Electron kinetics description with combined stochastic and Ohmic Fokker-Planck heating operator

L.L. Alves[1], Ts. V. Tsankov[2] and U. Czarnetzki[3]

[1] Instituto de Plasmas e Fusão Nuclear, Instituto Superior Técnico, Universidade de Lisboa, Lisboa, PORTUGAL

[2] LPP, École Polytechnique, Sorbonne Université, Institut Polytechnique de Paris, CNRS, FRANCE

[3] Ruhr University Bochum, Bochum, GERMANY

78th Annual Gaseous Electronics Conference (GEC), 13th-17th October 2025, COEX, Seoul, Republic of Korea

Abstract

The electron kinetics in low-temperature plasmas is commonly described by the electron Boltzmann equation (EBE), incorporating either Ohmic (collisional) or non-local (stochastic/collisionless) heating terms, depending on whether the plasma operates at high or low pressure, respectively.

At low pressures (~Pa), typical of ICPs, the electron energy relaxation length exceeds the characteristic system size. In such regimes, the electron population can be described by a non-local (global) distribution function that depends solely on the total energy. Furthermore, when energy transfer from the external field is confined to a small portion of the plasma volume, variations in the electron distribution function due to the plasma potential can be neglected, as the latter primarily affects the electron density.

Under these conditions, the electron kinetics can be modeled by an isotropic, non-local electron energy distribution function (EEDF), obtained by solving the EBE with:
(i) an energy gain term due to the action of an external electric field, and
(ii) energy loss terms due to collisions and transport to the reactor walls.

The energy gain term is described by a Fokker-Planck-type heating operator [1], which unifies the treatment of non-local stochastic heating, resulting from the spatial distribution of the electric field, and local Ohmic heating in the presence of electron-neutral collisions.

In this work, we present initial results from the numerical implementation of this approach in the LisbOn Kinetics Boltzmann and Chemistry solver (LoKI-B+C) [2]. We apply the description to a low-pressure argon ICP (p=0.2-1.5 Pa, n_e~10^{16} m^{-3}), assuming an exponential decay of the electric field away from the antenna (13.56 MHz).

The self-consistent solution provides the EEDF, the densities of the main argon species, the reduced electric-field intensity, the sheath potential, the skin depth, and the power coupled to the plasma. Simulation results are compared with experimental measurements of the EEDF obtained using electrostatic probes.

Excitation conditions (typical of RF ICPs)

Plasma reactor with heating zone close to antenna

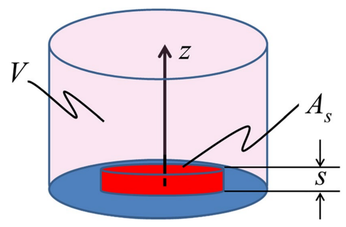
Transversal EM field, propagating in the z-direction ("infinitely" long: L≫R)

Exponential decay with distance from the antenna

$$\vec{E}(\vec{r},t) = \vec{E}(\vec{r})\cos(\omega_0 t) \equiv \vec{E}_0(\vec{r}_\parallel)\zeta(z)\cos(\omega_0 t)$$
$$\vec{E}(\vec{r}) = E_0(\vec{r}_\parallel)e^{-z/s}\vec{e}_x \ (z \geq 0)$$
$$\zeta^{\exp}(z) = e^{-|z|/s}$$

Oscillating frequency ~10MHz

$$\omega_{p,i} < \omega_0 < \omega_{p,e}$$



$$V = A_s L \simeq A_\parallel L$$

Collisional skin depth

$$s = \frac{c}{\omega_{pe}}\frac{\sqrt{2[1+(\nu_m/\omega_0)^2]}}{\sqrt{1+\sqrt{1+(\nu_m/\omega_0)^2}}}$$

The electron Boltzmann equation

$$\frac{\partial \underset{\sim}{f}(\vec{r},u)}{\partial t} = \left.\frac{\partial \underset{\sim}{f}(\vec{r},u)}{\partial t}\right|_{\text{heat}} + \left.\frac{\partial \underset{\sim}{f}(\vec{r},u)}{\partial t}\right|_{\text{surf}} + \left.\frac{\partial \underset{\sim}{f}(\vec{r},u)}{\partial t}\right|_{\text{coll}}$$

$$\underset{\sim}{f}(\vec{r},u) = \underset{\sim}{f}(u-\Phi(\vec{r})) \simeq \frac{n_e(\vec{r})}{n_0} f(u)$$

$$\int_0^\infty f(u)\sqrt{u}du = 1$$

Volume average EBE

$$-\sqrt{\frac{m_e}{2e}}\frac{1}{N}\frac{d\left(G_{\text{el}}(u)+G_{ee}(u)\right)}{du} - \sqrt{\frac{m_e}{2e}}\frac{1}{N}\frac{n_e^{\text{edge}}}{\langle n_e(\vec{r})\rangle_V}\frac{dG_{\text{FPH}}(u)}{du} - \frac{n_e^{\text{edge}}}{\langle n_e(\vec{r})\rangle_V}\frac{1}{4NL'}(u-\Phi)\Theta(u-\Phi)f(u) + S(u) = 0$$

$$L' \equiv \frac{V}{A_{\text{wall}}} = \frac{1}{2}\frac{L}{L/R+1}$$

Electron surface losses

Volume average of the electron - particle flux across the reactor walls (for electrons with kinetic energies above the sheath potential $\phi = V_s$)

Electron collisional losses

Isotropic component of the classical two-term development of the electron distribution function

Volume average ion particle-balance equation

$$\frac{n_e^{\text{edge}}}{\langle n_e(\vec{r})\rangle_V}\sqrt{\frac{2e}{m_e}}\frac{1}{4NL'}\int_\Phi^\infty (u-\Phi)f(u)du = \int_0^\infty \sqrt{\frac{2e}{m_e}}S(u)du = \frac{\nu_{\text{ion}}}{N}$$

Volume average electron particle-balance equation

$$\frac{1}{V}\int_{\vec{r}} \vec{\nabla}_{\vec{r}}\cdot\vec{\Gamma}_i d^3r = \frac{1}{V}\int_{\vec{r}} n_e(\vec{r})\nu_{\text{ion}}\, d^3r$$
$$\Rightarrow \frac{\langle n_e(\vec{r})\rangle_V}{n_e^{\text{edge}}} \simeq \frac{A_{\text{wall}}}{V}\frac{c_i}{\nu_{\text{ion}}} = \frac{1}{NL'}\frac{c_i}{\nu_{\text{ion}}/N}$$

Fokker-Planck heating operator (FPHO)

Fokker-Planck approach [1]

- Markov processes (no memory effects)
  - resulting from interactions of electrons with the electric field
  - yielding continuous changes in the electron velocities (energies)
  - described at kinetic level using distribution functions
- Master energy balance equation converted into differential eq. system
  Kramers-Moyal expansion; exact truncation to second-order
- Expansion considering a velocity-dependent collision frequency
- Averaging over the solid angle in velocity space
  vanishing of the drift term; preservation of the diffusion term
- Solution of the Langevin equation
  to obtain the average quadratic variations of the velocity from the equation of motion of particles in an external field

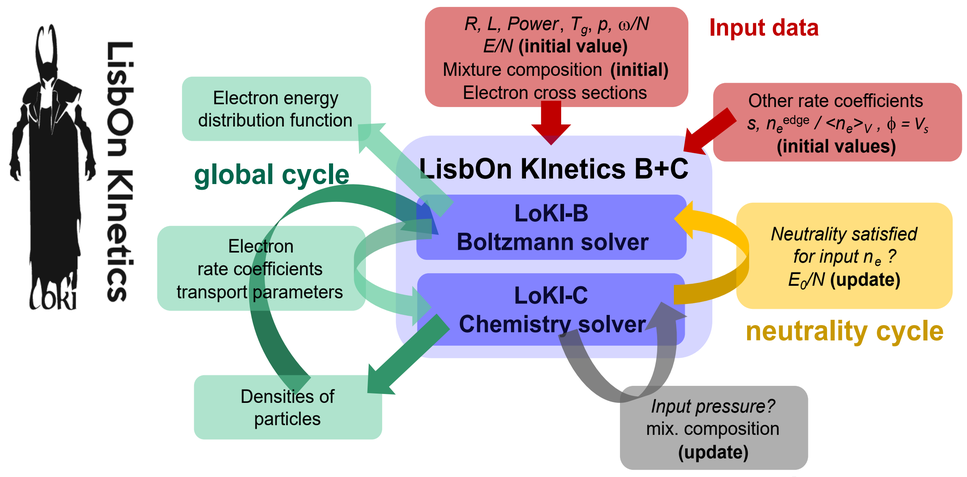
$$G_{\text{FPH}}(u) = -N\frac{2m_e}{6e}\frac{\omega_0}{N}\langle\nu_E^2\rangle_V^{\text{dist}}\left[u^{3/2}g_{OS}^{\text{dist}}(u)\frac{df(u)}{du}\right]$$

Ohmic-Stochastic (OS) coupling function and volume-averaged electron velocity

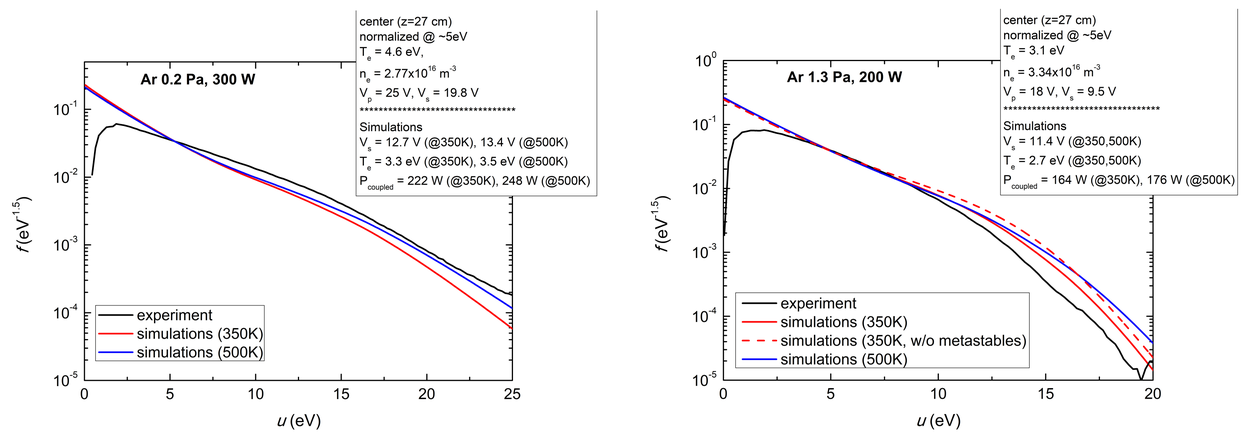
$$g_{OS}^{\text{dist-exp}}(\nu) = \frac{s\omega_0}{2\nu_{th}}\left\{\frac{3s\omega_0}{\nu}\left[\left(\frac{1}{2}+\frac{s^2(\omega_0^2-\nu_c^2(\nu))}{\nu^2}\right)\ln\left(\frac{\omega_0^2+(\nu/s+\nu_c(\nu))^2}{\omega_0^2+\nu_c^2(\nu)}\right)-1\right]\right.$$
$$\left.+6\frac{s^3\omega_0^2\nu_c(\nu)}{\nu^3}\left[\frac{\nu}{s\omega_0}-2\arctan\left(\frac{\nu\omega_0}{s(\omega_0^2+\nu_c(\nu)(\nu/s+\nu_c(\nu)))}\right)\right]\right\}$$

$$\langle\nu_E^2\rangle_V^{\text{dist-exp}} = \left(\frac{e}{m_e\omega_0}\right)^2\frac{1}{V}\frac{\nu_{th}}{\omega_0}\int_{r_\parallel} E_0^2(\vec{r}_\parallel)d^2r_\parallel$$
$$\simeq \left(\frac{e}{m_e\omega_0}\right)^2\frac{A_\parallel}{V}\frac{\nu_{th}}{\omega_0}E_0^2\,,$$

Numerical implementation in LoKI-B+C



Results for argon @ R = 0.25m, L = 0.5m



References

[1] U Czarnetzki and LL Alves, *Rev. Mod. Plasma Phys.* **6** 31 (2022)

[2] A Tejero *et al.*, *Plasma Sources Sci. Technol.* **28** 043001 (2019)

Acknowledgements

This work was partially funded by FCT - Fundação para a Ciência e a Tecnologia under projects UIDB/50010/2023, UID/PRR/50010/2025, LA/P/0061/2020 and 2022.04128.PTDC.

The authors thank A Tejero-Del-Caz for assistance in the numerical implementation of the formulation.

Fundação para a Ciência e a Tecnologia

Download this poster

Discussion and outlook

- General good agreement between simulations and measurements
- Relevant effect of the gas temperature in the simulations
- Stepwise excitation/ionization from metastables influence results
- Simulation results yield V_S ~ 5 T_e (for Argon), as predicted by phenomenological sheath theory
- Simulation results yield a Paschen-like curve for power vs. pressure
- Self-consistent LoKI-B+C simulations using either the OS-FPHO or its Ohmic limit yield very different E_0/N's to preserve the power gained from the electric field and satisfy neutrality.
- Ohmic calculations imposing the same E_0/N value obtained with OS-FPHO lead to disparate results for EEDF, whose differences attenuate with increasing pressure.
- Future work will focus on
  - Analysing simulation results for various ion transport models
  - Numerical implementation

Pushing the validity limits of FPHO